

Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 1 (Group)

香港数学竞赛 (2001 – 2002)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 假设曲线  $x^2 + 3y^2 = 12$  及直线  $mx + y = 16$  只相交于一点。若  $a = m^2$ ，求  $a$  的值。

Assume that the curve  $x^2 + 3y^2 = 12$  and the straight line  $mx + y = 16$  intersect at only one point.

If  $a = m^2$ , find the value of  $a$ .

$a =$

2. 已知  $x + y = 1$  及  $x^2 + y^2 = 2$ 。若  $x^3 + y^3 = b$ ，求  $b$  的值。

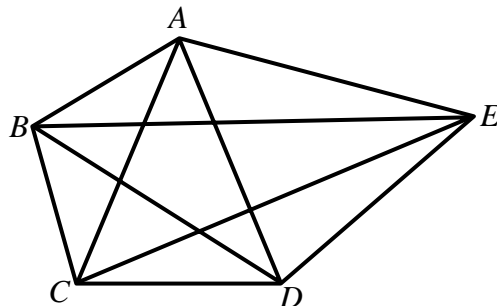
It is given that  $x + y = 1$  and  $x^2 + y^2 = 2$ . If  $x^3 + y^3 = b$ , find the value of  $b$ .

$b =$

3. 在下图中， $AC = AD = AE = ED = DB$  及  $\angle BEC = c^\circ$ 。已知  $\angle BDC = 26^\circ$  及  $\angle ADB = 46^\circ$ ，求  $c$  的值。

In the following figure,  $AC = AD = AE = ED = DB$  and  $\angle BEC = c^\circ$ . Given that  $\angle BDC = 26^\circ$  and  $\angle ADB = 46^\circ$ , find the value of  $c$ .

$c =$



4. 已知  $4\cos^4\theta + 5\sin^2\theta - 4 = 0$ ，其中  $0^\circ < \theta < 360^\circ$ 。若  $\theta$  的最大值为  $d$ ，求  $d$  的值。

It is given that  $4\cos^4\theta + 5\sin^2\theta - 4 = 0$ , where  $0^\circ < \theta < 360^\circ$ . If the maximum value of  $\theta$  is  $d$ , find the value of  $d$ .

$d =$
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Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 2 (Group)

香港数学竞赛 (2001 – 2002)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知三角形三边的长分别为 6、8 和 10。若这三角形的面积为  $a$ ，求  $a$  的值。

It is given that the lengths of the sides of a triangle are 6, 8 and 10. If the area of the triangle is  $a$ , find the value of  $a$ .

$a =$

2. 已知  $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$ 。若  $f(4) = b$ ，求  $b$  的值。

Given that  $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$  and  $f(4) = b$ , find the value of  $b$ .

$b =$

3. 已知  $2002^2 - 2001^2 + 2000^2 - 1999^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2 = c$ ，求  $c$  的值。

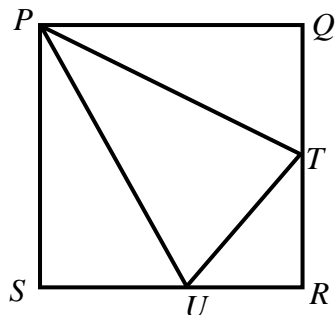
Given that  $2002^2 - 2001^2 + 2000^2 - 1999^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2 = c$ , find the value of  $c$ .

$c =$

4.  $PQRS$  为一正方形,  $PTU$  为一等腰三角形及  $\angle TPU = 30^\circ$ 。  $T$  及  $U$  分别为  $QR$  及  $RS$  上的点。  $\Delta PTU$  之面积为 1。若正方形  $PQRS$  之面积为  $d$ , 求  $d$  的值。

$PQRS$  is a square,  $PTU$  is an isosceles triangle, and  $\angle TPU = 30^\circ$ . Points  $T$  and  $U$  lie on  $QR$  and  $RS$  respectively. The area of  $\Delta PTU$  is 1. If the area of  $PQRS$  is  $d$ , find the value of  $d$ .

$d =$



Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 3 (Group)

香港数学竞赛 (2001 – 2002)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若  $\frac{2002^3 + 4 \times 2002^2 + 6006}{2002^2 + 2002} = a$ ，求  $a$  的值。

If  $\frac{2002^3 + 4 \times 2002^2 + 6006}{2002^2 + 2002} = a$ , find the value of  $a$ .

$a =$

2. 已知  $x$  和  $y$  为两实数且满足关系  $y = \frac{x}{2x-1}$ 。若  $\frac{1}{x^2} + \frac{1}{y^2}$  的最小值为  $b$ ，求  $b$  的值。

It is given that the real numbers  $x$  and  $y$  satisfy the relation  $y = \frac{x}{2x-1}$ . If the minimum value of  $\frac{1}{x^2} + \frac{1}{y^2}$  is  $b$ , find the value of  $b$ .

$b =$

3. 从 50 个正整数 1, 2, 3 ... 50 中任意抽两个不同的数。已知两数之和不少于 50。若抽取这两数共有  $c$  种取法，求  $c$  的值。

Suppose two different numbers are chosen randomly from the 50 positive integers 1, 2, 3, ..., 50, and the sum of these two numbers is not less than 50. If the number of ways of choosing these two numbers is  $c$ , find the value of  $c$ .

$c =$

4. 已知  $x - y = 1 + \sqrt{5}$ ， $y - z = 1 - \sqrt{5}$ 。若  $x^2 + y^2 + z^2 - xy - yz - zx = d$ ，求  $d$  的值。

Given that  $x - y = 1 + \sqrt{5}$ ,  $y - z = 1 - \sqrt{5}$ . If  $x^2 + y^2 + z^2 - xy - yz - zx = d$ , find the value of  $d$ .

$d =$

Hong Kong Mathematics Olympiad (2001 – 2002)

Final Event 4 (Group)

香港数学竞赛 (2001 – 2002)

决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若  $a$  是 2002 的所有正因子之和，求  $a$  的值。

If  $a$  is the sum of all the positive factors of 2002, find the value of  $a$ .

$a =$

2. 设  $x > 0$ ,  $y > 0$  且  $\sqrt{x}(\sqrt{x} + \sqrt{y}) = 3\sqrt{y}(\sqrt{x} + 5\sqrt{y})$ 。若  $b = \frac{2x + \sqrt{xy} + 3y}{x + \sqrt{xy} - y}$ , 求  $b$  的值。

It is given that  $x > 0$ ,  $y > 0$  and  $\sqrt{x}(\sqrt{x} + \sqrt{y}) = 3\sqrt{y}(\sqrt{x} + 5\sqrt{y})$ . If  $b = \frac{2x + \sqrt{xy} + 3y}{x + \sqrt{xy} - y}$ , find

the value of  $b$ .

$b =$

3. 若方程  $||x-2|-1|=c$  只有 3 个整数解，求  $c$  的值。

Given that the equation  $||x-2|-1|=c$  has only 3 integral solutions, find the value of  $c$ .

$c =$

4. 若  $d$  是方程  $\frac{1}{2}\left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2 + 2\right) + 2\right] + 2\right\} = 2$  的正实数解，求  $d$  的值。

If  $d$  is the positive real root of the equation  $\frac{1}{2}\left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2 + 2\right) + 2\right] + 2\right\} = 2$ , find the value of  $d$ .

$d =$